

90 Minutes—No Calculator

Note: In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e).

1. Which of the following defines a function f for which $f(-x) = -f(x)$?

(A) $f(x) = x^2$ (B) $f(x) = \sin x$ (C) $f(x) = \cos x$
(D) $f(x) = \log x$ (E) $f(x) = e^x$

2. $\ln(x-2) < 0$ if and only if

(A) $x < 3$ (B) $0 < x < 3$ (C) $2 < x < 3$
(D) $x > 2$ (E) $x > 3$

3. If $\begin{cases} f(x) = \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & \text{for } x \neq 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{7}{5}$

4. $\int_0^8 \frac{dx}{\sqrt{1+x}} =$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

5. If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

(A) -2 (B) 0 (C) 2 (D) 4 (E) not defined

6. What is $\lim_{h \rightarrow 0} \frac{8\left(\frac{1}{2} + h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?
- (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist.
- (E) It cannot be determined from the information given.
-
7. For what value of k will $x + \frac{k}{x}$ have a relative maximum at $x = -2$?
- (A) -4 (B) -2 (C) 2 (D) 4 (E) None of these
-
8. If $p(x) = (x + 2)(x + k)$ and if the remainder is 12 when $p(x)$ is divided by $x - 1$, then $k =$
- (A) 2 (B) 3 (C) 6 (D) 11 (E) 13
-
9. When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is
- (A) $\frac{1}{4\pi}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\pi}$ (D) 1 (E) π
-
10. The set of all points (e^t, t) , where t is a real number, is the graph of $y =$
- (A) $\frac{1}{e^x}$ (B) $e^{\frac{1}{x}}$ (C) $x e^{\frac{1}{x}}$ (D) $\frac{1}{\ln x}$ (E) $\ln x$
-
11. The point on the curve $x^2 + 2y = 0$ that is nearest the point $\left(0, -\frac{1}{2}\right)$ occurs where y is
- (A) $\frac{1}{2}$ (B) 0 (C) $-\frac{1}{2}$ (D) -1 (E) none of the above

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12. If $f(x) = \frac{4}{x-1}$ and $g(x) = 2x$, then the solution set of $f(g(x)) = g(f(x))$ is

- (A) $\left\{\frac{1}{3}\right\}$ (B) $\{2\}$ (C) $\{3\}$ (D) $\{-1, 2\}$ (E) $\left\{\frac{1}{3}, 2\right\}$

13. The region bounded by the x -axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \leq x \leq k$ is three times the area of the region for $k \leq x \leq \frac{\pi}{2}$, then $k =$

- (A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$
 (D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

14. If the function f is defined by $f(x) = x^5 - 1$, then f^{-1} , the inverse function of f , is defined by $f^{-1}(x) =$

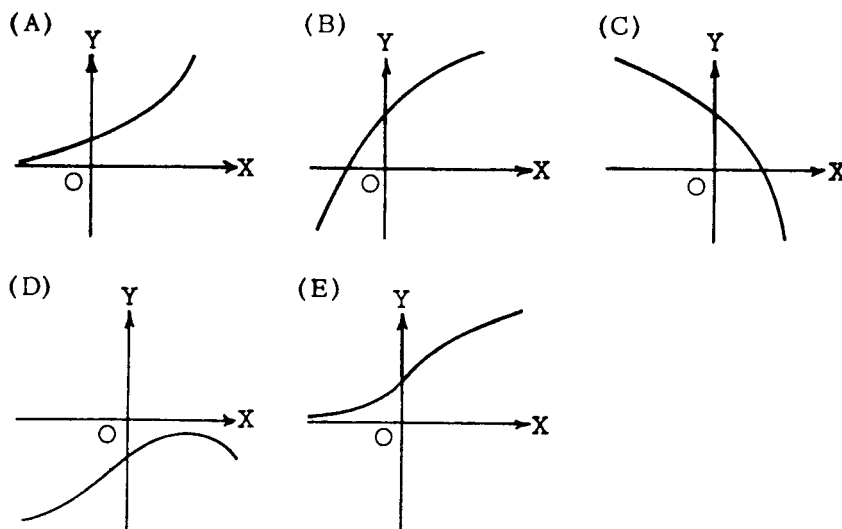
- (A) $\frac{1}{\sqrt[5]{x+1}}$ (B) $\frac{1}{\sqrt[5]{x-1}}$ (C) $\sqrt[5]{x-1}$
 (D) $\sqrt[5]{x} - 1$ (E) $\sqrt[5]{x+1}$

15. If $f'(x)$ and $g'(x)$ exist and $f'(x) > g'(x)$ for all real x , then the graph of $y = f(x)$ and the graph of $y = g(x)$

- (A) intersect exactly once.
 (B) intersect no more than once.
 (C) do not intersect.
 (D) could intersect more than once.
 (E) have a common tangent at each point of intersection.

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16. If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x , which of the following could be part of the graph of $y = f(x)$?



17. The graph of $y = 5x^4 - x^5$ has a point of inflection at

- (A) $(0,0)$ only (B) $(3,162)$ only (C) $(4,256)$ only
(D) $(0,0)$ and $(3,162)$ (E) $(0,0)$ and $(4,256)$

18. If $f(x) = 2 + |x - 3|$ for all x , then the value of the derivative $f'(x)$ at $x = 3$ is

- (A) -1 (B) 0 (C) 1 (D) 2 (E) nonexistent

19. A point moves on the x -axis in such a way that its velocity at time t ($t > 0$) is given by $v = \frac{\ln t}{t}$.
At what value of t does v attain its maximum?

- (A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$
(E) There is no maximum value for v .

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20. An equation for a tangent to the graph of $y = \arcsin \frac{x}{2}$ at the origin is
- (A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$ (D) $y = 0$ (E) $\pi x - 2y = 0$
-
21. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?
- (A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.
-
22. $\frac{d}{dx}(\ln e^{2x}) =$
- (A) $\frac{1}{e^{2x}}$ (B) $\frac{2}{e^{2x}}$ (C) $2x$ (D) 1 (E) 2
-
23. The area of the region bounded by the curve $y = e^{2x}$, the x -axis, the y -axis, and the line $x = 2$ is equal to
- (A) $\frac{e^4}{2} - e$ (B) $\frac{e^4}{2} - 1$ (C) $\frac{e^4}{2} - \frac{1}{2}$
(D) $2e^4 - e$ (E) $2e^4 - 2$
-
24. If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x ?
- (A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

25. A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x -axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

- (A) is independent of m .
 - (B) increases as m increases.
 - (C) decreases as m increases.
 - (D) decreases as m increases when $m < \frac{1}{2}$; increases as m increases when $m > \frac{1}{2}$.
 - (E) increases as m increases when $m < \frac{1}{2}$; decreases as m increases when $m > \frac{1}{2}$.
-

26. $\int_0^1 \sqrt{x^2 - 2x + 1} \, dx$ is

- (A) -1
 - (B) $-\frac{1}{2}$
 - (C) $\frac{1}{2}$
 - (D) 1
 - (E) none of the above
-

27. If $\frac{dy}{dx} = \tan x$, then $y =$

- (A) $\frac{1}{2} \tan^2 x + C$
 - (B) $\sec^2 x + C$
 - (C) $\ln|\sec x| + C$
 - (D) $\ln|\cos x| + C$
 - (E) $\sec x \tan x + C$
-

28. The function defined by $f(x) = \sqrt{3} \cos x + 3 \sin x$ has an amplitude of

- (A) $3 - \sqrt{3}$
- (B) $\sqrt{3}$
- (C) $2\sqrt{3}$
- (D) $3 + \sqrt{3}$
- (E) $3\sqrt{3}$

29. $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} dx =$

- (A) $\ln \sqrt{2}$ (B) $\ln \frac{\pi}{4}$ (C) $\ln \sqrt{3}$ (D) $\ln \frac{\sqrt{3}}{2}$ (E) $\ln e$

30. If a function f is continuous for all x and if f has a relative maximum at $(-1, 4)$ and a relative minimum at $(3, -2)$, which of the following statements must be true?

- (A) The graph of f has a point of inflection somewhere between $x = -1$ and $x = 3$.
(B) $f'(-1) = 0$
(C) The graph of f has a horizontal asymptote.
(D) The graph of f has a horizontal tangent line at $x = 3$.
(E) The graph of f intersects both axes.

31. If $f'(x) = -f(x)$ and $f(1) = 1$, then $f(x) =$

- (A) $\frac{1}{2}e^{-2x+2}$ (B) e^{-x-1} (C) e^{1-x} (D) e^{-x} (E) $-e^x$

32. If a, b, c, d , and e are real numbers and $a \neq 0$, then the polynomial equation $ax^7 + bx^5 + cx^3 + dx + e = 0$ has

- (A) only one real root.
(B) at least one real root.
(C) an odd number of nonreal roots.
(D) no real roots.
(E) no positive real roots.

33. What is the average (mean) value of $3t^3 - t^2$ over the interval $-1 \leq t \leq 2$?

- (A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

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34. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = \frac{1}{x} + k$ (where k takes all real values)?
- (A) $y = -x$ (B) $y = -x^2$ (C) $y = -\frac{1}{3}x^3$ (D) $y = \frac{1}{3}x^3$ (E) $y = \ln x$
-
35. At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?
- (A) 32 (B) 48 (C) 64 (D) 96 (E) 192
-
36. The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is
- (A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24
-
37. Which is the best of the following polynomial approximations to $\cos 2x$ near $x = 0$?
- (A) $1 + \frac{x}{2}$ (B) $1 + x$ (C) $1 - \frac{x^2}{2}$ (D) $1 - 2x^2$ (E) $1 - 2x + x^2$
-
38. $\int \frac{x^2}{e^{x^3}} dx =$
- (A) $-\frac{1}{3} \ln e^{x^3} + C$ (B) $-\frac{e^{x^3}}{3} + C$ (C) $-\frac{1}{3e^{x^3}} + C$
- (D) $\frac{1}{3} \ln e^{x^3} + C$ (E) $\frac{x^3}{3e^{x^3}} + C$
-
39. If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?
- (A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $\frac{2}{e}$ (E) $\sec^2 e$

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40. If n is a non-negative integer, then $\int_0^1 x^n dx = \int_0^1 (1-x)^n dx$ for

- (A) no n (B) n even, only (C) n odd, only
(D) nonzero n , only (E) all n

41. If $\begin{cases} f(x) = 8 - x^2 & \text{for } -2 \leq x \leq 2, \\ f(x) = x^2 & \text{elsewhere,} \end{cases}$ then $\int_{-1}^3 f(x) dx$ is a number between

- (A) 0 and 8 (B) 8 and 16 (C) 16 and 24 (D) 24 and 32 (E) 32 and 40

42. What are all values of k for which the graph of $y = x^3 - 3x^2 + k$ will have three distinct x -intercepts?

- (A) All $k > 0$ (B) All $k < 4$ (C) $k = 0, 4$ (D) $0 < k < 4$ (E) All k

43. $\int \sin(2x+3) dx =$

- (A) $\frac{1}{2} \cos(2x+3) + C$ (B) $\cos(2x+3) + C$ (C) $-\cos(2x+3) + C$
(D) $-\frac{1}{2} \cos(2x+3) + C$ (E) $-\frac{1}{5} \cos(2x+3) + C$

44. The fundamental period of the function defined by $f(x) = 3 - 2 \cos^2 \frac{\pi x}{3}$ is

- (A) 1 (B) 2 (C) 3 (D) 5 (E) 6

45. If $\frac{d}{dx}(f(x)) = g(x)$ and $\frac{d}{dx}(g(x)) = f(x^2)$, then $\frac{d^2}{dx^2}(f(x^3)) =$

- (A) $f(x^6)$ (B) $g(x^3)$ (C) $3x^2 g(x^3)$
(D) $9x^4 f(x^6) + 6x g(x^3)$ (E) $f(x^6) + g(x^3)$

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|-------|-------|
| 1. B | 24. C |
| 2. C | 25. A |
| 3. B | 26. C |
| 4. D | 27. C |
| 5. E | 28. C |
| 6. B | 29. A |
| 7. D | 30. E |
| 8. B | 31. C |
| 9. C | 32. B |
| 10. E | 33. A |
| 11. B | 34. D |
| 12. A | 35. A |
| 13. C | 36. B |
| 14. E | 37. D |
| 15. B | 38. C |
| 16. B | 39. D |
| 17. B | 40. E |
| 18. E | 41. D |
| 19. C | 42. D |
| 20. A | 43. D |
| 21. B | 44. C |
| 22. E | 45. D |
| 23. C | |

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|-------|-------|
| 1. C | 24. C |
| 2. E | 25. A |
| 3. B | 26. C |
| 4. D | 27. C |
| 5. E | 28. D |
| 6. B | 29. C |
| 7. D | 30. D |
| 8. C | 31. C |
| 9. D | 32. B |
| 10. A | 33. A |
| 11. B | 34. D |
| 12. E | 35. A |
| 13. C | 36. B |
| 14. D | 37. D |
| 15. B | 38. A |
| 16. B | 39. D |
| 17. B | 40. E |
| 18. E | 41. D |
| 19. C | 42. B |
| 20. A | 43. E |
| 21. B | 44. E |
| 22. E | 45. E |
| 23. D | |

1. B Sine is the only odd function listed. $\sin(-x) = -\sin(x)$.

2. C $\ln t < 0$ for $0 < t < 1 \Rightarrow \ln(x-2) < 0$ for $2 < x < 3$.

3. B Need to have $\lim_{x \rightarrow 2} f(x) = f(2) = k$.

$$\begin{aligned} k &= \lim_{x \rightarrow 2} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} = \lim_{x \rightarrow 2} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} \cdot \frac{\sqrt{2x+5} + \sqrt{x+7}}{\sqrt{2x+5} + \sqrt{x+7}} \\ &= \lim_{x \rightarrow 2} \frac{2x+5 - (x+7)}{x-2} \cdot \frac{1}{\sqrt{2x+5} + \sqrt{x+7}} = \lim_{x \rightarrow 2} \frac{1}{\sqrt{2x+5} + \sqrt{x+7}} = \frac{1}{6} \end{aligned}$$

4. D $\int_0^8 \frac{dx}{\sqrt{1+x}} = 2\sqrt{1+x} \Big|_0^8 = 2(3-1) = 4$

5. E Using implicit differentiation, $6x + 2xy' + 2y + 2y \cdot y' = 0$. Therefore $y' = \frac{-2y-6x}{2x+2y}$.

When $x = 1$, $3 + 2y + y^2 = 2 \Rightarrow 0 = y^2 + 2y + 1 = (y+1)^2 \Rightarrow y = -1$

Therefore $2x + 2y = 0$ and so $\frac{dy}{dx}$ is not defined at $x = 1$.

6. B This is the derivative of $f(x) = 8x^8$ at $x = \frac{1}{2}$

$$f'\left(\frac{1}{2}\right) = 64\left(\frac{1}{2}\right)^7 = \frac{1}{2}$$

7. D With $f(x) = x + \frac{k}{x}$, we need $0 = f'(-2) = 1 - \frac{k}{4}$ and so $k = 4$. Since $f''(-2) < 0$ for $k = 4$, f does have a relative maximum at $x = -2$.

8. B $p(x) = q(x)(x-1) + 12$ for some polynomial $q(x)$ and so $12 = p(1) = (1+2)(1+k) \Rightarrow k = 3$

9. C $A = \pi r^2$, $\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$ and from the given information in the problem $\frac{dA}{dt} = 2 \frac{dr}{dt}$.

So, $2 \frac{dr}{dt} = 2\pi r \cdot \frac{dr}{dt} \Rightarrow r = \frac{1}{\pi}$

10. E $x = e^y \Rightarrow y = \ln x$

11. B Let L be the distance from $\left(x, -\frac{x^2}{2}\right)$ and $\left(0, -\frac{1}{2}\right)$.

$$L^2 = (x-0)^2 + \left(\frac{x^2}{2} - \frac{1}{2}\right)^2$$

$$2L \cdot \frac{dL}{dx} = 2x + 2\left(\frac{x^2}{2} - \frac{1}{2}\right)(x)$$

$$\frac{dL}{dx} = \frac{2x + 2\left(\frac{x^2}{2} - \frac{1}{2}\right)(x)}{2L} = \frac{2x + x^3 - x}{2L} = \frac{x^3 + x}{2L} = \frac{x(x^2 + 1)}{2L}$$

$$\frac{dL}{dx} < 0 \text{ for all } x < 0 \text{ and } \frac{dL}{dx} > 0 \text{ for all } x > 0, \text{ so the minimum distance occurs at } x = 0.$$

The nearest point is the origin.

12. A $\frac{4}{2x-1} = 2\left(\frac{4}{x-1}\right) \Rightarrow x-1 = 4x-2; x = \frac{1}{3}$

13. C $\int_{-\pi/2}^k \cos x \, dx = 3 \int_k^{\pi/2} \cos x \, dx; \sin k - \sin\left(-\frac{\pi}{2}\right) = 3\left(\sin \frac{\pi}{2} - \sin k\right)$
 $\sin k + 1 = 3 - 3 \sin k; 4 \sin k = 2 \Rightarrow k = \frac{\pi}{6}$

14. E $y = x^5 - 1$ has an inverse $x = y^5 - 1 \Rightarrow y = \sqrt[5]{x+1}$

15. B The graphs do not need to intersect (e.g. $f(x) = -e^{-x}$ and $g(x) = e^{-x}$). The graphs could intersect (e.g. $f(x) = 2x$ and $g(x) = x$). However, if they do intersect, they will intersect no more than once because $f(x)$ grows faster than $g(x)$.

16. B $y' > 0 \Rightarrow y$ is increasing; $y'' < 0 \Rightarrow$ the graph is concave down. Only B meets these conditions.

17. B $y' = 20x^3 - 5x^4$, $y'' = 60x^2 - 20x^3 = 20x^2(3-x)$. The only sign change in y'' is at $x = 3$. The only point of inflection is $(3, 162)$.

18. E There is no derivative at the vertex which is located at $x = 3$.

19. C $\frac{dv}{dt} = \frac{1 - \ln t}{t^2} > 0$ for $0 < t < e$ and $\frac{dv}{dt} < 0$ for $t > e$, thus v has its maximum at $t = e$.

20. A $y(0) = 0$ and $y'(0) = \frac{\frac{1}{2}}{\sqrt{1 - \frac{x^2}{4}}}\bigg|_{x=0} = \frac{1}{\sqrt{4 - x^2}}\bigg|_{x=0} = \frac{1}{2}$. The tangent line is
 $y = \frac{1}{2}x \Rightarrow x - 2y = 0$.

21. B $f'(x) = 2x - 2e^{-2x}$, $f'(0) = -2$, so f is decreasing

22. E $\ln e^{2x} = 2x \Rightarrow \frac{d}{dx}(\ln e^{2x}) = \frac{d}{dx}(2x) = 2$

23. C $\int_0^2 e^{2x} dx = \frac{1}{2} e^{2x} \bigg|_0^2 = \frac{1}{2}(e^4 - 1)$

24. C $y = \ln \sin x$, $y' = \frac{\cos x}{\sin x} = \cot x$

25. A $\int_m^{2m} \frac{1}{x} dx = \ln x \bigg|_m^{2m} = \ln(2m) - \ln(m) = \ln 2$ so the area is independent of m .

26. C $\int_0^1 \sqrt{x^2 - 2x + 1} dx = \int_0^1 |x - 1| dx = \int_0^1 -(x - 1) dx = -\frac{1}{2}(x - 1)^2 \bigg|_0^1 = \frac{1}{2}$

Alternatively, the graph of the region is a right triangle with vertices at $(0,0)$, $(0,1)$, and $(1,0)$.

The area is $\frac{1}{2}$.

27. C $\int \tan x dx = \int \frac{\sin x}{\cos x} dx = -\ln |\cos x| + C = \ln |\sec x| + C$

28. C $\sqrt{3} \cos x + 3 \sin x$ can be thought of as the expansion of $\sin(x + y)$. Since $\sqrt{3}$ and 3 are too large for values of $\sin y$ and $\cos y$, multiply and divide by the result of the Pythagorean Theorem used on those values, i.e. $2\sqrt{3}$. Then

$$\begin{aligned}\sqrt{3} \cos x + 3 \sin x &= 2\sqrt{3} \left(\frac{\sqrt{3}}{2\sqrt{3}} \cos x + \frac{3}{2\sqrt{3}} \sin x \right) = 2\sqrt{3} \left(\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \right) \\ &= 2\sqrt{3} (\sin y \cos x + \cos y \sin x) = 2\sqrt{3} \sin(y + x)\end{aligned}$$

where $y = \sin^{-1}\left(\frac{1}{2}\right)$. The amplitude is $2\sqrt{3}$.

Alternatively, the function $f(x)$ is periodic with period 2π . $f'(x) = -\sqrt{3} \sin x + 3 \cos x = 0$ when $\tan x = \sqrt{3}$. The solutions over one period are $x = \frac{\pi}{3}, \frac{4\pi}{3}$. Then $f\left(\frac{\pi}{3}\right) = 2\sqrt{3}$ and $f\left(\frac{4\pi}{3}\right) = -2\sqrt{3}$. So the amplitude is $2\sqrt{3}$.

29. A $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} dx = \ln(\sin x) \Big|_{\pi/4}^{\pi/2} = \ln 1 - \ln \frac{1}{\sqrt{2}} = \ln \sqrt{2}$
30. E Because f is continuous for all x , the Intermediate Value Theorem implies that the graph of f must intersect the x -axis. The graph must also intersect the y -axis since f is defined for all x , in particular, at $x = 0$.
31. C $\frac{dy}{dx} = -y \Rightarrow y = ce^{-x}$ and $1 = ce^{-1} \Rightarrow c = e$; $y = e \cdot e^{-x} = e^{1-x}$
32. B If $a < 0$ then $\lim_{x \rightarrow -\infty} y = \infty$ and $\lim_{x \rightarrow \infty} y = -\infty$ which would mean that there is at least one root. If $a > 0$ then $\lim_{x \rightarrow -\infty} y = -\infty$ and $\lim_{x \rightarrow \infty} y = \infty$ which would mean that there is at least one root. In both cases the equation has at least one root.
33. A $\frac{1}{3} \int_{-1}^2 3t^3 - t^2 dt = \frac{1}{3} \left(\frac{3}{4} t^4 - \frac{1}{3} t^3 \right) \Big|_{-1}^2 = \frac{1}{3} \left(\left(12 - \frac{8}{3} \right) - \left(\frac{3}{4} - \frac{1}{3} \right) \right) = \frac{11}{4}$
34. D $y' = -\frac{1}{x^2}$, so the desired curve satisfies $y' = x^2 \Rightarrow y = \frac{1}{3} x^3 + C$

35. A $a(t) = 24t^2$, $v(t) = 8t^3 + C$ and $v(0) = 0 \Rightarrow C = 0$. The particle is always moving to the right, so distance $= \int_0^2 8t^3 dt = 2t^4 \Big|_0^2 = 32$.
36. B $y = \sqrt{4 + \sin x}$, $y(0) = 2$, $y'(0) = \frac{\cos 0}{2\sqrt{4 + \sin 0}} = \frac{1}{4}$. The linear approximation to y is $L(x) = 2 + \frac{1}{4}x$. $L(1.2) = 2 + \frac{1}{4}(1.2) = 2.03$
37. D All options have the same value at $x = 0$. We want the one that has the same first and second derivatives at $x = 0$ as $y = \cos 2x$: $y'(0) = -2 \sin 2x \Big|_{x=0} = 0$ and $y''(0) = -4 \cos 2x \Big|_{x=0} = -4$.
For $y = 1 - 2x^2$, $y'(0) = -4x \Big|_{x=0} = 0$ and $y''(0) = -4$ and no other option works.
38. C $\int \frac{x^2}{e^{-x^3}} dx = -\frac{1}{3} \int e^{-x^3} (-3x^2 dx) = -\frac{1}{3} e^{-x^3} + C = -\frac{1}{3e^{-x^3}} + C$
39. D $x = e \Rightarrow v = 1$, $u = 0$, $y = 0$; $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx} = (\sec^2 u) \left(1 + \frac{1}{v^2}\right) \left(\frac{1}{x}\right) = (1)(2)(e^{-1}) = \frac{2}{e}$
40. E One solution technique is to evaluate each integral and note that the value is $\frac{1}{n+1}$ for each.
Another technique is to use the substitution $u = 1 - x$; $\int_0^1 (1-x)^n dx = \int_1^0 u^n (-du) = \int_0^1 u^n du$.
Integrals do not depend on the variable that is used and so $\int_0^1 u^n du$ is the same as $\int_0^1 x^n dx$.
41. D $\int_{-1}^3 f(x) dx = \int_{-1}^2 (8 - x^2) dx + \int_2^3 x^2 dx = \left(8x - \frac{1}{3}x^3\right) \Big|_{-1}^2 + \frac{1}{3}x^3 \Big|_2^3 = 27 \frac{1}{3}$
42. D $y = x^3 - 3x^2 + k$, $y' = 3x^2 - 6x = 3x(x - 2)$. So f has a relative maximum at $(0, k)$ and a relative minimum at $(2, k - 4)$. There will be 3 distinct x -intercepts if the maximum and minimum are on the opposite sides of the x -axis. We want $k - 4 < 0 < k \Rightarrow 0 < k < 4$.
43. D $\int \sin(2x + 3) dx = -\frac{1}{2} \cos(2x + 3) + C$

44. C Since $\cos 2A = 2\cos^2 A - 1$, we have $3 - 2\cos^2 \frac{\pi x}{3} = 3 - (1 + \cos \frac{2\pi x}{3})$ and the latter

expression has period $\frac{2\pi}{\left(\frac{2\pi}{3}\right)} = 3$

45. D Let $y = f(x^3)$. We want y'' where $f'(x) = g(x)$ and $f''(x) = g'(x) = f'(x^2)$

$$y = f(x^3)$$

$$y' = f'(x^3) \cdot 3x^2$$

$$y'' = 3x^2 \left(f''(x^3) \cdot 3x^2 \right) + f'(x^3) \cdot 6x$$

$$= 9x^4 f''(x^3) + 6x f'(x^3) = 9x^4 f'((x^3)^2) + 6x g(x^3) = 9x^4 f'(x^6) + 6x g(x^3)$$